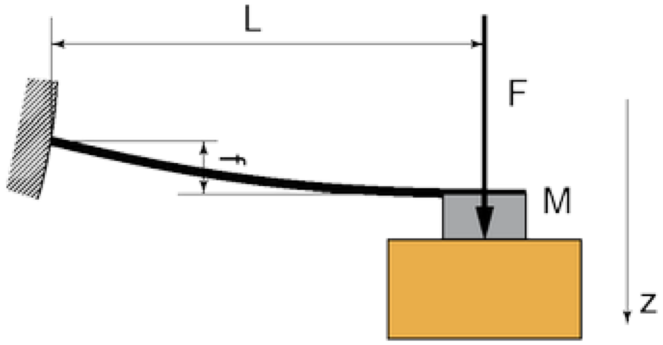
EE-580 Space Mechanisms

Exercise 4.2 - Stiffness and stresses

v.02

Problem Statement

An elastic beam keep a mass M in contact with a surface according to the following scheme



The beam has the following dimensions:

- Thickness t = 0.5 mm
- Width w = 5 mm
- Material: CuBe (E = 113 GPa)
- Length of the beam L = 35 mm
- Deflection f = 2 mm

Mass being pushed against the surface M = 2 g (grams)

Questions

1. What is the stiffness of the beam?
2. What is the force F of the mass on the surface?
3. Calculate the maximum stress in the beam.
4. Where is the maximum stress in the beam located?
5. Which acceleration shall be applied to the system in the z direction in order to remove the contact between the mass and the surface?
6. Taking into account a maximum allowable stress for CuBe of 400 MPa, what is the margin of safety (MOS) for this system, with the applied deflection f (FOS = 1)?

Definition

The Margin of Safety (MOS) for a stress σ compared to a reference stress $\sigma0$ with a Factor of Safety FOS is:

$$\text{MOS} = \frac{\sigma 0}{\sigma \cdot \text{FOS}} - 1$$

Solution

Mobile mass [kg]:

In[∘]:= $M = 2 \times 10^{-3};$

Elastic beam:

Length [m]

In[∘]:= $L = 35 \times 10^{-3};$

Width [m]:

In[∘]:= $w = 5 \times 10^{-3};$

Thickness [m]:

In[∘]:= $t = 0.5 \times 10^{-3};$

Deflection [m]:

In[∘]:= $f = 2 \times 10^{-3};$

Material: CuBe
Copper beryllium (CuBe) is a precipitation hardening alloy, which can attain very high strength, depending on the heat treatment. It’s properties may hence vary a lot depending on the heat treatment. The values used here are more “as-procured” values, without hardening. Datasheet from the manufacturer shall be used for real applications.

Young’s Modulus [Pa]:

In[∘]:= $E_{CuBe} = 113 \times 10^{9};$

Ultimate Tensile Strength [Pa]:

In[∘]:= $\sigma_u = 500 \times 10^{6};$

Yield strength [Pa]:

In[∘]:= $\sigma_y = 400 \times 10^{6};$

Calculation

Area moment of inertia of the axis [m ^{4}]:

In[ ]:= $I_{beam} = \frac{w\,t^3}{12}$

Out[ ]= 5.20833×10^{-14}

1. Stiffness of the beam [N/m]:

In[ ]:= $k = \frac{3\,E_{CuBe}\,I_{beam}}{L^3}$

Out[ ]= 411.808

2. Force applied on the surface [N]:

In[ ]:= $F = k\,f$

Out[ ]= 0.823615

3. Maximum stress [Pa]

In[ ]:= $\sigma_{max} = \frac{k\,f\,L}{2\,I_{beam}}\,t$

Out[ ]= 1.38367×10^{8}

4. Location of the maximum stress

The maximum stress is located at the attachment side of the beam

5. Contact loss acceleration [m/s ^{2}]

From second Newton's law:

In[ ]:= $a = \frac{k\,f}{M}$

Out[ ]= 411.808

In [g's]:

In[ ]:= a = a / 9.81

Out[ ]= 41.9783

6. Margin of Safety

Factor of Safety (for preloaded device):

In[ ]:= FOS = 1

Out[ ]= 1

Margin of Safety

In[ ]:= $MOS_y = \frac{\sigma_y}{\sigma_{max}\ FOS} - 1$

Out[ ]=

1.89086

In[ ]:= $MOS_u = \frac{\sigma_u}{\sigma_{max}\ FOS} - 1$

Out[ ]=

2.61357